Phase transitions in a system of indirect magnetoexcitons in coupled quantum wells at high magnetic field: the role of disorder

Oleg L. Berman[1], Yurii E. Lozovik[2], David W. Snoke[3], and Rob D. Coalson[1]
[1] *Department of Chemistry, University of Pittsburgh, Pittsburgh, PA 15260, USA*
[2] *Institute of Spectroscopy, Russian Academy of Sciences, 142190 Troitsk, Moscow Region, Russia*
[3] *Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260 USA*

Collective properties of a quasi-two-dimensional ($2D$) system of spatially indirect magnetoexcitons in coupled quantum wells (CQW) in high magnetic field H were analyzed in the presence of disorder. The Hamiltonian of the dilute gas of magnetoexcitons with dipole-dipole repulsion in a random field has been reduced to the Hamiltonian of a dilute gas of dipolar excitons without an applied magnetic field, but in an H-dependent effective random field and having an effective mass of magnetoexciton which is a function of the magnetic field and parameters of the CQW. For $2D$ magnetoexcitonic systems, the increase of the magnetic field H and the interwell distance D is found to increase the effective renormalized random field parameter Q and suppress the superfluid density n_s and the temperature of the Kosterlitz-Thouless transition T_c. It is shown that in the presence of the disorder there is a quantum transition to the superfluid state at zero temperature $T = 0$ with respect to the magnetic field H and the parameters of the disorder. There is no superfluidity at any exciton density in the presence of the disorder at sufficiently large magnetic field H or sufficiently large disorder.

Key words: coupled quantum wells (CQW), nanostructures, superfluidity, magnetoexciton, Bose-Einstein condensation of magnetoexcitons.
PACS numbers: 71.35.Ji, 71.35.Lk, 71.35.-y

Indirect excitons in coupled quantum wells (CQW) both with and without magnetic fields (H) have been the subject of recent experimental investigations (see Fig.1).[1−7] These systems are of interest, in particular, in connection with the possibility of Bose-Einstein condensation and superfluidity of indirect excitons or electron-hole pairs, which would manifest itself in the CQW as persistent electrical currents in each well and also through coherent optical properties and Josephson phenomena.[8−13] In high magnetic fields ($H > 7$T) two-dimensional (2D) excitons survive in a substantially wider temperature region, as the exciton binding energies increase with magnetic field.[14−22] The theory of magnetoexcitonic systems developed to date has not taken into account the influence of random field on the phase transitions, which is created by impurities and boundary irregularities of the quantum wells. In real experiments, however, disorder plays a very important role. Although the inhomogeneous broadening linewidth of typical GaAs-based samples has been improved from around 20 meV to less than 1 meV,[5] this disorder energy is still not much smaller than the exciton-exciton repulsion energy. (At a typical exciton density of 10^{10} cm^{-2}, the interaction energy of the excitons is approximately 1 meV.[4]) On the other hand, the typical disorder energy of 1 meV is low compared to the typical exciton binding energy of 5 meV.

In the present Letter we study superfluidity in a "dirty" system of indirect excitons in strong magnetic field H. We reduce the problem of magnetoexcitons in random fields to the problem of excitons at $H = 0$ and in a renormalized random field depending on H. We analyze the dependence of Kosterlitz-Thouless transition[23] temperature and superfluid density on magnetic field.

The total Hamiltonian $\hat{H}$ describing 2D spatially separated electrons (e) and holes (h) in a perpendicular magnetic field in the presence of the external field has the form:

$$\begin{aligned}\hat{H} &= \int d\mathbf{R}\int d\mathbf{r}\Big[\hat{\psi}^{\dagger}(\mathbf{R},\mathbf{r})\\ &\quad\left(\frac{1}{2m_e}\left(-i\nabla_e + e\mathbf{A}_e\right)^2 + \frac{1}{2m_h}\left(-i\nabla_h - e\mathbf{A}_h\right)^2\right.\\ &- \left.\frac{e^2}{\epsilon\sqrt{(\mathbf{r}_e-\mathbf{r}_h)^2+D^2}} + V_e(\mathbf{r}_e) + V_h(\mathbf{r}_h)\right)\hat{\psi}(\mathbf{R},\mathbf{r})\Big]\\ &+ \frac{1}{2}\int d\mathbf{R}_1\int d\mathbf{r}_1\int d\mathbf{R}_2\int d\mathbf{r}_2\hat{\psi}^{\dagger}(\mathbf{R}_1,\mathbf{r}_1)\hat{\psi}^{\dagger}(\mathbf{R}_2,\mathbf{r}_2)\\ &\quad\big(U^{ee}(\mathbf{r}_{e1}-\mathbf{r}_{e2}) + U^{hh}(\mathbf{r}_{h1}-\mathbf{r}_{h2}) + U^{eh}(\mathbf{r}_{e1}-\mathbf{r}_{h2})\\ &+ U^{he}(\mathbf{r}_{h1}-\mathbf{r}_{e2})\big)\,\hat{\psi}(\mathbf{R}_2,\mathbf{r}_2)\hat{\psi}(\mathbf{R}_1,\mathbf{r}_1). \qquad (1)\end{aligned}$$

Here $\hat{\psi}^{\dagger}(\mathbf{R},\mathbf{r})$ and $\hat{\psi}(\mathbf{R},\mathbf{r})$ are the creation and annihilation operators for magnetoexcitons; $\mathbf{r}_e$ and $\mathbf{r}_h$ are electron and hole locations along quantum wells, correspondingly; $\mathbf{A}_e$, $\mathbf{A}_h$ are the vector potentials at the electron and hole location, respectively; $V_e(\mathbf{r}_e)$ and $V_h(\mathbf{r}_h)$ represent the external fields acting on electron and hole, respectively (we use units $c = \hbar = 1$); D is the distance between electron and hole quantum wells; e is the charge of an electron; ϵ is the dielectric constant. We use below the coordinates of the magnetoexciton center of mass $\mathbf{R} = (m_e\mathbf{r}_e + m_h\mathbf{r}_h)/(m_e + m_h)$ and the

internal exciton coordinates $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$. The cylindrical gauge for vector-potential is used: $\mathbf{A}_{e,h} = \frac{1}{2}\mathbf{H} \times \mathbf{r}_{e,h}$. U^{ee}, U^{hh}, U^{eh} and U^{he} are the two-particle potentials of the electron-electron, hole-hole, electron-hole and hole-electron interaction, respectively, between electrons or holes from different pairs: $U^{ee}(\mathbf{r}_{e1} - \mathbf{r}_{e2}) = e^2/(\epsilon|\mathbf{r}_{e1} - \mathbf{r}_{e2}|)$; $U^{hh}(\mathbf{r}_{h1} - \mathbf{r}_{h2}) = e^2/(\epsilon|\mathbf{r}_{h1} - \mathbf{r}_{h2}|)$; $U^{eh}(\mathbf{r}_{e1} - \mathbf{r}_{h2}) = -e^2/(\epsilon\sqrt{|\mathbf{r}_{e1} - \mathbf{r}_{h2}|^2 + D^2})$; $U^{he}(\mathbf{r}_{h1} - \mathbf{r}_{e2}) = -e^2/(\epsilon\sqrt{|\mathbf{r}_{h1} - \mathbf{r}_{e2}|^2 + D^2})$.

Note that the exciton *magnetic* momentum[24] $\hat{\mathbf{P}} = -i\nabla_e - i\nabla_h + e(\mathbf{A}_e - \mathbf{A}_h) - e\mathbf{H} \times (\mathbf{r}_e - \mathbf{r}_h)$ is a conserved quantity for an isolated exciton in a magnetic field without any external field ($V_e(\mathbf{r}_e) = V_h(\mathbf{r}_h) = 0$). The eigenfunctions of the Hamiltonian of a single isolated magnetoexciton without any random field ($V_e(\mathbf{r}_e) = V_h(\mathbf{r}_h) = 0$), which are also the eigenfunctions of the magnetic momentum $\mathbf{P}$, have the following form (see Refs. [14,24]):

$$\Psi_{k\mathbf{P}}(\mathbf{R}, \mathbf{r}) = \exp\left\{ i\mathbf{R}\left(\mathbf{P} + \frac{e}{2}\mathbf{H} \times \mathbf{R}\right) + i\gamma\frac{\mathbf{P}\mathbf{r}}{2}\right\}\Phi_k(\mathbf{P}, \mathbf{r}), \quad (2)$$

where $\Phi_k(\mathbf{P}, \mathbf{r})$ is a function of internal coordinates $\mathbf{r}$; $\mathbf{P}$ is the eigenvalue of magnetic momentum; k represents the quantum numbers of exciton internal motion. In high magnetic fields $k = (n_L, m)$, where $n_L = min(n_1, n_2)$, $m = |n_1 - n_2|$, and $n_{1(2)}$ are Landau quantum numbers for e and h[14,18]; $\gamma = (m_h - m_e)/(m_h + m_e)$.

We further expand the magnetoexciton field operators in a single magnetoexciton basis set $\Psi_{k\mathbf{P}}(\mathbf{R}, \mathbf{r})$: $\hat{\psi}^\dagger(\mathbf{R}, \mathbf{r}) = \sum_{k\mathbf{P}} \Psi^*_{k\mathbf{P}}(\mathbf{R}, \mathbf{r})\hat{a}^\dagger_{k\mathbf{P}}$; $\hat{\psi}(\mathbf{R}, \mathbf{r}) = \sum_{k\mathbf{P}} \Psi_{k\mathbf{P}}(\mathbf{R}, \mathbf{r})\hat{a}_{k\mathbf{P}}$, where $\hat{a}^\dagger_{k\mathbf{P}}$ and $\hat{a}_{k\mathbf{P}}$ are the corresponding creation and annihilation operators of a magnetoexciton in $(k, \mathbf{P})$ space.

We consider the case of strong magnetic field, when we neglect in Eq. (2) the transitions between different Landau levels of the magnetoexciton caused by scattering by the slowly changing in space potential $V_e(\mathbf{r}_e) + V_h(\mathbf{r}_h)$. We also neglect nondiagonal matrix elements of the Coulomb interaction between a paired electron and hole. The region of applicability of these two assumptions is defined by the inequalities[25] $\omega_c \gg E_b$, $\omega_c \gg \sqrt{\left\langle V^2_{e(h)}\right\rangle_{av}}$, where $\omega_c = eH/m_{e-h}$, $m_{e-h} = m_e m_h/(m_e + m_h)$ is the exciton reduced mass in the quantum well plane; E_b is the magnetoexciton binding energy in an ideal "pure" system as as a function of magnetic field H and the distance between electron and hole quantum wells D: $E_b \sim e^2/\epsilon r_H\sqrt{\pi/2}$ at $D \ll r_H$ and $E_b \sim e^2/\epsilon D$ at $D \gg r_H$ ($r_H = (eH)^{-1/2}$ is the magnetic length).[14,18] Here $\langle \ldots \rangle_{av}$ denotes averaging over the fluctuations of the random field.

In a strong magnetic field at low densities ($n \ll r_H^{-2}$) indirect magnetoexcitons repel as parallel dipoles,[19], and we have for the pair interaction potential:

$$\hat{U}(|\mathbf{R}_1 - \mathbf{R}_2|) \equiv \hat{U}^{ee} + \hat{U}^{hh} + \hat{U}^{eh} + \hat{U}^{he} \simeq \frac{e^2 D^2}{\epsilon|\mathbf{R}_1 - \mathbf{R}_2|^3}. \quad (3)$$

Now we substitute the expansions for the field creation and annihilation operators into the total Hamiltonian Eq. (1) and obtain the effective Hamiltonian in terms of creation and annihilation operators in $\mathbf{P}$ space. In high magnetic field, when the typical interexciton interaction $D^2 n^{-\frac{3}{2}} \ll \omega_c$, one can ignore transitions between Landau levels and consider only the states corresponding to the lowest Landau level $m = n_L = 0$. Using the orthonormality of the functions $\Phi_{mn}(\mathbf{0}, \mathbf{r})$ we obtain the effective Hamiltonian $\hat{H}_{\rm eff}$ in strong magnetic fields. Since a typical value of r is r_H, and $P \ll 1/r_H$, in this approximation the effective Hamiltonian $\hat{H}_{\rm eff}$ in the magnetic momentum representation P in the subspace the lowest Landau level $m = n_L = 0$ has the same form (compare with Ref.[26]) as for two-dimensional boson system without a magnetic field, but with the magnetoexciton magnetic mass m_H (which depends on H and D; see below) instead of the exciton mass ($M = m_e + m_h$), magnetic momenta instead of ordinary momenta and renormalized random field (for the lowest Landau level we denote the spectrum of the single exciton $\varepsilon_0(P) \equiv \varepsilon_{00}(\mathbf{P})$):

$$\hat{H}_{\rm eff} = \sum_{\mathbf{P}} \varepsilon_0(P)\hat{a}^\dagger_{\mathbf{P}}\hat{a}_{\mathbf{P}} + \sum_{\mathbf{P},\mathbf{P}'} \left\langle \mathbf{P}' \left| \hat{V} \right| \mathbf{P} \right\rangle \hat{a}^\dagger_{\mathbf{P}'}\hat{a}_{\mathbf{P}} + \frac{1}{2} \sum_{\mathbf{P}_1,\mathbf{P}_2,\mathbf{P}_3,\mathbf{P}_4} \left\langle \mathbf{P}_1, \mathbf{P}_2 \left| \hat{U} \right| \mathbf{P}_3, \mathbf{P}_4 \right\rangle \hat{a}^\dagger_{\mathbf{P}_1}\hat{a}^\dagger_{\mathbf{P}_2}\hat{a}_{\mathbf{P}_3}\hat{a}_{\mathbf{P}_4}, \quad (4)$$

where $\hat{V} = \hat{V}_e + \hat{V}_h$. The dispersion relation $\varepsilon_0(P)$ of an isolated magnetoexciton on the lowest Landau level is a quadratic function at the small magnetic momenta under consideration, $\varepsilon_0(\mathbf{P}) \approx P^2/(2m_H)$, where m_H is the effective *magnetic* mass of a magnetoexciton in the lowest Landau level, dependent on H and the distance D between $e-$ and $h-$ layers (see Ref.[18]). In strong magnetic fields at $D \gg r_H$ the exciton magnetic mass is $m_H \approx D^3\epsilon/(e^2 r_H^4)$[18].

The matrix element $\left\langle \mathbf{P}_1, \mathbf{P}_2 \left| \hat{U} \right| \mathbf{P}_3, \mathbf{P}_4 \right\rangle$ is the Fourier transform of the pair interaction potential (Eq. (3)). In the strong magnetic field limit, using for $\Phi_k(\mathbf{0}, \mathbf{r})$ the internal wavefunction of the magnetoexciton at the lowest Landau level $\Phi_{k=0}(\mathbf{0}, \mathbf{r})$,[18] we obtain the matrix element of the external potential $V_{e,h}(\mathbf{r})$ connecting the states $\langle k = 0, \mathbf{P} |$ and $\langle k = 0, \mathbf{P}' |$:

$$\left\langle \mathbf{P}' \mid \hat{V}_{e,h}(\mathbf{r}) \mid \mathbf{P} \right\rangle = \frac{1}{S}\exp\left(-(\mathbf{P}' - \mathbf{P})^2\frac{r_H^2}{4}\right) \times V_{e,h}(\mathbf{P}' - \mathbf{P})\exp\left(\pm\frac{ir_H^2}{2H}\mathbf{H}\cdot(\mathbf{P} \times \mathbf{P}')\right), \quad (5)$$

where $V_{e,h}(\mathbf{P}' - \mathbf{P})$ is the Fourier transform of $V_{e,h}(\mathbf{r})$.

Thus, the effective Hamiltonian $\hat{H}_{\rm eff}$ Eq. (4) corresponds to excitons with renormalized dispersion law $\varepsilon_0(P)$ and effective random field

$$V_{\rm eff}(\mathbf{R}) = \frac{1}{\pi r_H^2}\int \exp\left(-\frac{(\mathbf{R} - \mathbf{r})^2}{r_H^2}\right) [V_e(\mathbf{r}) + V_h(\mathbf{r})]\, d\mathbf{r}. \quad (6)$$

The approach is valid, if the correlation length L of random potential ($V(\mathbf{r}_e, \mathbf{r}_h)$) is much greater than the magnetoexciton mean size[25] $r_{exc} = r_H$: ($L \gg r_H$) or $r_{exc}\sqrt{\langle \nabla V^2\rangle_{av}} \ll E_b$, and it holds for the strong magnetic field, when $r_{exc} = r_H = (eH)^{-1/2}$, and $E_b \sim e^2/\epsilon D$ at $D \gg r_H$[18,25].

The interaction between an spatially indirect exciton in coupled quantum wells and a random field, induced by fluctuations in widths of electron and hole quantum wells, has the form[25] $V(\mathbf{r}_e, \mathbf{r}_h) = \alpha_e[\xi_1(\mathbf{r}_e) - \xi_2(\mathbf{r}_e)] + \alpha_h[\xi_3(\mathbf{r}_h) - \xi_4(\mathbf{r}_h)]$, where $\alpha_{e,h} = \partial E_{e,h}^{(0)}/\partial d_{e,h}$, $d_{e,h}$ is the average widths of the electron,hole quantum wells, while $E_{e,h}^{(0)}$ are the lowest levels of the electron and hole in the conduction and valence bands, and ξ_1 and ξ_2 (ξ_3 and ξ_4) are fluctuations in the widths of the electron (hole) wells on the upper and lower interfaces, respectively. We assume that fluctuations on different interfaces are statistically independent, whereas fluctuations of a specific interface are characterized by Gaussian correlation function $\langle \xi_i(\mathbf{r}_1)\xi_j(\mathbf{r}_2)\rangle = g_i\delta_{ij}\delta(\mathbf{r}_2 - \mathbf{r}_1)$, $\langle \xi_i(\mathbf{r})\rangle = 0$, where g_i is proportional to the squared amplitude of the ith interface fluctuation[25]. This is possible if the distance D between the electron and hole quantum wells is larger than the amplitude of fluctuations on the nearest surfaces.

We consider the characteristic length of the random field potential L to be much shorter than the average distance between excitons $r_s \sim 1/\sqrt{\pi n}$ ($L \ll 1/\sqrt{\pi n}$, where n is the total exciton density) similar to Ref. [26]. Since the effective Hamiltonian $\hat{H}_{\rm eff}$ of the system of indirect "dirty" magnetoexcitons at small momenta is exactly identical to the Hamiltonian of indirect "dirty" excitons without magnetic field but with magnetic mass m_H and effective random field $V_{\rm eff}$ instead of $M = m_e + m_h$ and $V_e(\mathbf{r}) + V_h(\mathbf{r})$, respectively, we can use the expressions for the ladder approximation Green's function[27], collective spectrum, normal and superfluid density and the temperature of Kosterlitz-Thouless phase transition[23] for the "dirty"[26] system without magnetic field.

The spectrum of interacting excitons has the form (cf. Ref. [26])

$$\varepsilon(p) = \sqrt{\left(p^2/(2m_H) + \sqrt{\mu^2 - Q^2}\right)^2 - (\mu^2 - Q^2)},$$

and for small momenta $p \ll \sqrt{2m_H\mu}$ the excitation spectrum is acoustic $\varepsilon(p) = c_s p$, where $c_s = \sqrt{\sqrt{\mu^2 - Q^2}/m_H}$ is the velocity of sound. The chemical potential μ in the ladder approximation has the form[26] $\mu = 8\pi n/\left[2m_H \log\left(\epsilon^2/(8\pi n m_H^2 e^4 D^4)\right)\right]$. In the weak-scattering limit we use the second-order Born approximation for the random field parameter Q similar to Refs. [26], and for small frequencies and momenta, which mostly contribute to the ladder approximation Green's function, approximate $Q(\mathbf{p}, \omega)$ by $Q(\mathbf{p} = \mathbf{0}, \omega = 0)$

$$Q(\mathbf{p}, \omega) \ = \ Q = \frac{\alpha_e^2(g_1 + g_2) + \alpha_h^2(g_3 + g_4)}{64\pi^4} m_H. \quad (7)$$

The density of the superfluid component $n_s(T)$ can be obtained using the relation $n_s(T) = n - n_n(T)$, where $n_n(T)$ is the density of the normal component. The density of normal component n_n is (compare to Ref. [26]):

$$n_n = \frac{3\zeta(3)}{2\pi}\frac{T^3}{c_s^4(n, Q)m_H} + \frac{nQ}{2m_H c_s^2(n, Q)}. \quad (8)$$

From Eq. (8) we can see, that the random field decreases the density of the superfluid component.

In a 2D system, superfluidity appears below the Kosterlitz-Thouless transition temperature $T_c = \pi n_s/(2m_H)$[23], where only coupled vortices are present. Using the expression (8) for the density n_s of the superfluid component, we obtain an equation for the Kosterlitz-Thouless transition temperature T_c. Its solution is

$$T_c \ = \ \left[\left(1 + \sqrt{\frac{32}{27}\left(\frac{m_H T_c^0}{\pi n'}\right)^3 + 1}\right)^{1/3} - \left(\sqrt{\frac{32}{27}\left(\frac{m_H T_c^0}{\pi n'}\right)^3 + 1} - 1\right)^{1/3}\right]\frac{T_c^0}{2^{1/3}}. \quad (9)$$

Here T_c^0 is an auxiliary quantity, equal to the temperature at which the superfluid density vanishes in the mean-field approximation $n_s(T_c^0) = 0$, $T_c^0 = \left(2\pi n' c_s^4 m_H/(3\zeta(3))\right)^{1/3}$, $n' = n - nQ/(2m_H c_s^2)$.

Since in strong magnetic fields at $D \gg r_H$ the exciton magnetic mass is $m_H \approx D^3\epsilon/(e^2 r_H^4)$,[18] the superfluid density n_s and the temperature of the Kosterlitz-Thouless transition T_c decrease with increase of the magnetic field H, and the parameters of the random field α_e, α_h and g_i. Since in the "dirty" systems n_s and T_c decrease with the increase of effective random field Q (analogous to the case without magnetic field[26]), and in a strong magnetic field Q is proportional to m_H (Eq. (7)), an increase of the magnetic field H increases the effective renormalized random field Q, and thus suppresses the superfluid density n_s and the temperature of the Kosterlitz-Thouless transition T_c.

It follows from Eq. (8) that in the presence of disorder at $T = 0$ there is a quantum transition from the superfluid state to a Bose glass at a sufficiently large value of the magnetic field H and the parameters of the disorder α_e, α_h and g_i. While in the "pure" system at any magnetic field H at there is always a region in the density-temperature space, where the superfluidity occurs[19], in the presence of the disorder at sufficiently large magnetic field H or the parameters of the disorder α_e, α_h and g_i there is no superfluidity at any exciton density.

Note also that in a magnetic field the superfluid density n_s and the temperature of the Kosterlitz-Thouless transition T_c decrease when the separation between quantum wells D increases. As D increases, so do m_H[18] and thus Q (Eq. (7)): increasing either of these parameters

decreases n_s and T_c (Eqs. (8) and (9)). There is a competing influence, namely that increasing D increases the velocity of sound (since it increases the chemical potential of the dipole-dipole repulsion μ), which tends to increase n_s and T_c, but this is a logarithmic dependence. Thus, in a strong magnetic field the first two influences dominate, and n_s and T_c decrease with D. In the absence of a magnetic field, the first two influences are absent: in this case, the aforementioned logarithmic dependence dominates, causing n_s and T_c to increase with D.[26]

In the high magnetic field limit at high D, the effective random field is not small, and approaches which assume coupling with the random field to be much smaller than the dipole-dipole repulsion are not applicable. Note that in the present work the parameter Q/μ is not required to be very small, and our formulas for the superfluid density and Kosterlitz-Thouless temperature can be used in the regime of realistic experimental parameters taken from photoluminescence line broadening measurements[5].

Acknowledgements

O. L. B. wishes to thank the participants of the First International Conference on Spontaneous Coherence in Excitonic Systems (ICSCE) in Seven Springs PA for many useful and stimulating discussions. Yu. E. L. was supported by the INTAS grant. D. W. S. and R. D. C. have been supported by the National Science Foundation.

---

[1] L. V. Butov, A. Zrenner, G. Abstreiter, G. Bohm and G. Weimann, Phys. Rev. Lett. **73**, 304 (1994); L. V. Butov and A. I. Filin, Phys. Rev. **B 58**, 1980 (1998).

[2] S. Denev, V. Negoita, D. W. Snoke, B. Laikhtman, K. Eberl, and L. Pfeiffer, Phys. Rev. **B 66**, 205304 (2002).

[3] A. V. Larionov, V. B. Timofeev, J. Hvam, and K. Soerensen, Russ. JETP Lett. **75**, 200 (2002).

[4] For a general review of experiments in quantum wells, see chapter 10 of S. A. Moskalenko and D. W. Snoke, *Bose-Einstein Condensation of Excitons and Biexcitons and Coherent Nonlinear Optics with Excitons* (Cambridge University Press, New York, 2000).

[5] D. W. Snoke, S. Denev, Y. Liu, L. Pfeiffer and K. West, Nature **418**, 754 (2002).

[6] D. W. Snoke, Science **298**, 1368 (2002).

[7] L. V. Butov, C. W. Lai, A. L. Ivanov, A. C. Gossard, and D. S. Chemla, Nature **417**, 47 (2002); L. V. Butov, A. C. Gossard, and D. S. Chemla, Nature **418**, 751 (2002)).

[8] Yu. E. Lozovik and V. I. Yudson, Russ. JETP Lett. **22**, 26(1975); Russ. JETP **44**, 389 (1976); Yu. E. Lozovik and A. V. Poushnov, Phys. Lett. **A 228**, 399 (1997); Yu. E. Lozovik and I. V. Ovchinnikov, Phys. Rev. **B 66**, 075124 (2002).

[9] J. Zang, D. Schmeltzer and J. L. Birman, Phys. Rev. Lett. **71**, 773 (1993).

[10] Xu. Zhu, P. B. Littlewood, M. S. Hybertsen and T. M. Rice, Phys. Rev. Lett. **74**, 1633 (1995).

[11] S. Conti, G. Vignale and A. H. MacDonald, Phys. Rev. **B 57**, R6846 (1998).

[12] Yu. E. Lozovik and O. L. Berman, Russ. JETP Lett. **64**, 573 (1996); Russ. JETP **84**, 1027 (1997).

[13] Yu. E. Lozovik, O. L. Berman, and M. Willander, J. Phys.: Condens. Matter **14**, 12457 (2002).

[14] I. V. Lerner and Yu. E. Lozovik, Russ. JETP **51**, 588 (1980); Russ. JETP **53**, 763 (1981); A. B. Dzjubenko and Yu. E. Lozovik, Russ. Solid State Phys. **25**, 874 (1983); J. Phys. **A 24**, 415 (1991).

[15] D. Paquet, T. M. Rice, and K. Ueda, Phys. Rev. **B 32**, 5208 (1985).

[16] C. Kallin and B. I. Halperin, Phys. Rev. **B 30**, 5655 (1984); Phys. Rev. **B 31**, 3635 (1985).

[17] D. Yoshioka and A. H. MacDonald, J. Phys. Soc. of Japan, **59**, 4211 (1990).

[18] Yu. E. Lozovik and A. M. Ruvinsky, Russ. JETP **85**, 979 (1997).

[19] Yu. E. Lozovik, O. L. Berman and V. G. Tsvetus, Phys. Rev. **B 59**, 5627 (1999).

[20] M. A. Olivares-Robles and S. E. Ulloa, Phys. Rev. **B 64**, 115302 (2001).

[21] Yu. E. Lozovik, I. V. Ovchinnikov, S. Yu. Volkov, L. V. Butov, and D. S. Chemla, Phys. Rev. **B 65**, 235304 (2002).

[22] S. A. Moskalenko, M. A. Liberman, D. W. Snoke and V. V. Botan, Phys. Rev. **B 66**, 245316 (2002).

[23] J. M. Kosterlitz and D. J. Thouless, J. Phys. **C 6**, 1181 (1973); D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).

[24] L. P. Gor'kov and I. E. Dzyaloshinskii, Russ. JETP **26**, 449 (1968).

[25] Yu. E. Lozovik and A. M. Ruvinsky, Russ. JETP **87**, 788 (1998).

[26] O. L. Berman, Yu. E. Lozovik, D. W. Snoke and R. D. Coalson, Europhysics Letters (submitted)(cond-mat/0406587); Phys. Rev. **B** (submitted), (cond-mat/0408053).

[27] Yu. E. Lozovik and V. I. Yudson, Physica **A 93**, 493 (1978).

Caption to Figure 1

Fig.1 The geometry of the spatially separated electron (e) and hole (h) in coupled quantum wells in external magnetic (**H**) and electric (**E**) fields normal to quantum wells.

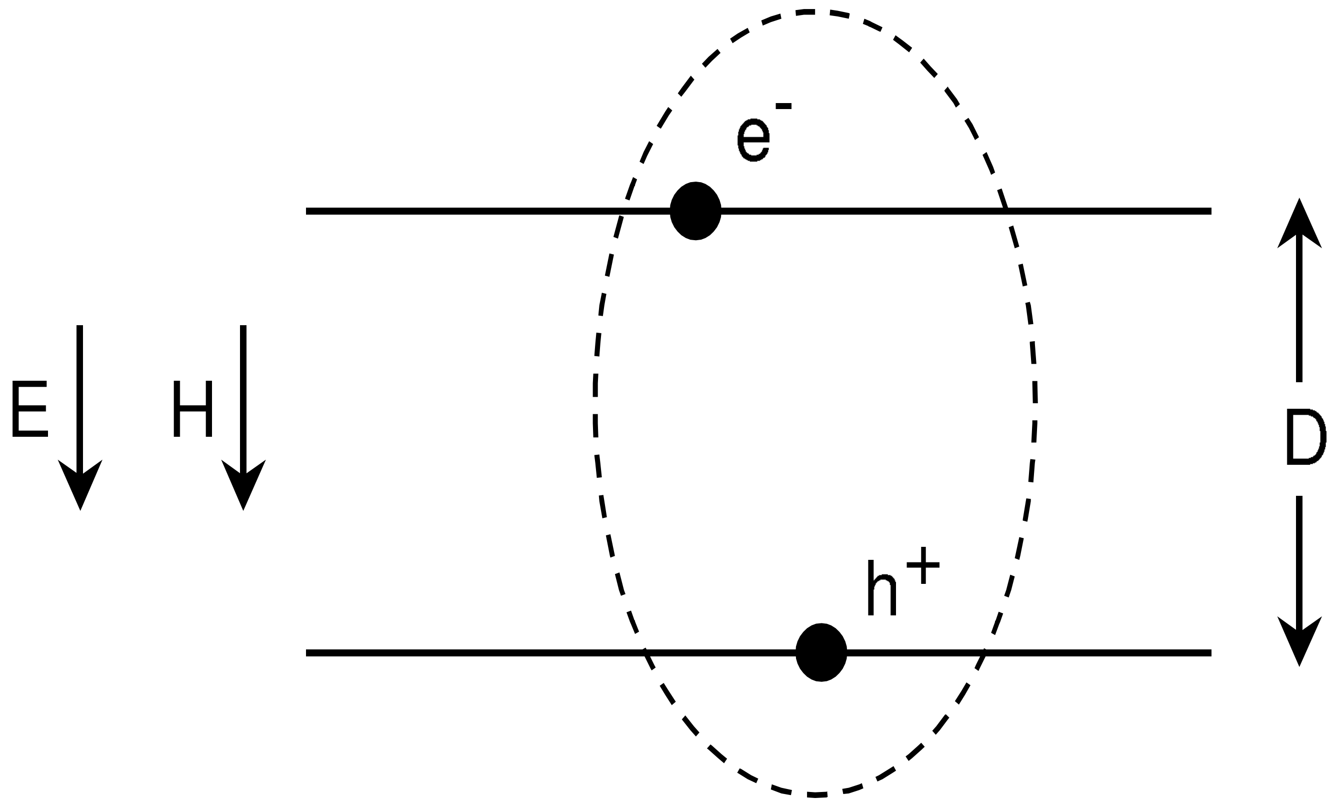

Fig. 1